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Optimal taxation with gradual learning of types

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Abstract

An important feature of life-cycle models is the presence of uncertainty regarding one's labor income. Yet this issue, long recognized in different areas, has not received enough attention in the optimal taxation literature. This paper is an attempt to fill this gap. We write a simple 3 period model where agents gradually learn their productivities. In a framework akin to Mirrlees' (1971) static one, we derive properties of optimal tax schedules and show that: i) if preferences are (weakly) separable, uniform taxation of goods is optimal, ii) if they are (strongly) separable capital income is to be taxed, and; iii) investments for retirement ought to be taxed at a lower rate than other forms of investment. **Keywords:** Optimal Taxation, Uncertain Productivity, Capital Income Taxation **JEL Classification:** H21, D82.

1 Introduction

Most economists would agree that, from the moment one enters the labor market to the moment he or she retires, there is a lot of uncertainty and a lot of learning regarding one's own earnings potential.

In fact, life-cycle models with this type of uncertainty have already been incorporated to asset pricing literature - Constantinides (2002), Harris and Willen (2001) are a few recent examples. Yet, this topic seems not to have received much attention in the optimal taxation literature.

As a first step towards filling this gap we write down, in this paper, a dynamic extension of Stiglitz's (1982) two types version of Mirrlees'(1971) classic optimal income taxation framework.

The paper describes a very stylized life-cycle model of optimal taxation where the information sets of agents evolve in a non-trivial fashion. That is, agents

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'know' more about their own productivity as they grow older. These changes are also translated into a gradual increase in heterogeniety among agents.

The intersection with Mirrlees (1971) - or its dicrete counterpart, Stiglits (1982) - comes form the fact that once an agent's productivity is realized, only she can observe it. That, is, there is private information on an agent's type.

A benevolent government which inhabits this economy tries to maximize agents' expected utility. Because an agents' productivity is her private information, the government has to rely on distortionary taxation to try to meet its goals. A precise mapping between the information structure and the set of insturments at the government's disposal is emphasized throughout the paper. This is another dimension at which we follow the tradition founded by Mirrlees (1971).

Agents are assumed to be identical when they are born. Were it not for the existence of such a first period we would be speaking of a utilitarian government. So the mapping is obvious and we shall no longer discuss the government objectives.

The interesting feature - the one that makes the problem non-standard - is the fact that agents learn their productivities as time evolves. The introduction of a non-trivial filtration[1], with decisions required to be adapted to it, creates a whole new set of interesting issues we shall address.

First, it is worth emphasizing that this intrdouces 'de facto' uncertainty to Mirrlees' model. Also, antecipation of deviation from prescribed behavior - that is antecipation of off-equilibrium choice - and not only the deviations themselves will matter for the design of an optimal tax system. A nice feature of our simpistic model is that it allows for isolating these effects.

We characterize the optimal tax system, emphasizing not only taxation on labor income but also on capital income. Supplementary commodity taxes are also discussed.

When it comes to labor income taxation, it is shown that efficient types are never distorted. However, if an agent receives a bad shock (type L), at any given period, she will always have her labor-leisure choice distorted. In particular, she will thereafter always face a positive marginal tax rate.

In an attempt to further characterize the optimal labor income tax schedule we explore tow possibilities. First, we propose allocations that are *isolated* in the following sense. The only influence of allocations of a node in the infromation tree on the allocations of a different node is through the channel of transfers in resources. Second, we construct an allocation that is *memoryless*, in that allocations are defined at each period with only the information arrived ate that period. We show that neither allocation is, in general, optimal.

We, then, explore aspects of intertemporal taxation. With a utility specification that exhibits separability between leisure and consumption, capital income is optimally taxed. This result should not be seen as a knife-edge case. In fact, strong complementarity in preferences are needed to break down this result. On

[1] We shall be calling non-trivial a filtration $\mathbb{F} \equiv \{\mathcal{F}\}_{t=0}^{T}$, such that there is at least one $t \in (1, ..., T)$ such that the σ-algebra $\mathcal{F}_t$ strictly contains $\mathcal{F}_{t-1}$.

the other hand, if a retirement period is included, retirees should *not* have their investments taxed under the same conditions.

Finally we show that (weak) separability on preferences render the uniform taxation result of Atkinson and Stiglitz (1976) optimal.

The rest of the paper is organized as follows. Section 2 presents the model economy we study. Optimal tax rates on labor income are derived in section 3, and on capital income in section 4. Supplementary commodity taxation is discussed in section 5. Section 6 concludes.

2 The Environment

The economy is populated by a continuum of 'ex-ante' identical agents with preferences defined over lotteries over sequences of labor, l, and consumption, x. Let $(x,l)_t \equiv \{x_s, l_s\}_{s=t}^{T}$. Absent any uncertainty, preferences would be represented by

$$U\left((x,l)_t\right) \equiv \sum\nolimits_{s=t}^{T} \delta^{s-t} u\left(x_s, l_s\right),$$

where u is a regular, concave and twice continuously differentiable function[2], with $u_x > 0, u_l < 0$.

Notice that we have not indexed preferences by types. At every moment and every state of nature, agents have identical preferences for bundles (or lotteries over bundles) of (x,l). Nonetheless, after the first period, heterogeneity will come up in preferences over sequences of output Y and income y, as a result of heterogeneity in productivity.

At each period an agent is endowed with a certain level of production capacity, or productivity, w. This assumption is only made to highlight some issues that are not directly dependent on heterogeneity but on the dynamic nature of the problem. They gradually learn about their productivities. As time elapses she is subject to shocks in her productivity that will gradually increase heterogeneity among agents in this economy.

Uncertainty arises in this model because paths of productivities are stochastic. That is in period t an agent doesn't know how productive she will be in period $t' > t$. Whether this form of heterogeneity is already present at birth (but not perceived by the agent) or it is developed along one's life is immaterial for the problem we study. What is relevant to point out is that it is neither a consequence of any (intensional) action by the agent nor may it be inferred by outcomes that are not measurable by the specific filtration we shall define.

As uncertainty is revealed, each agent gradually 'learns her type'. What makes the dynamic of this model more complex than the ones found in traditional optimal income taxation problems is the fact that decisions are made based on partial information. That is, we impose the natural requirement that,

[2] Subscripts here will always denote partial derivatives.

for each agent, sequences of choice variables be adapted to the filtration associated to her. This is a minimum requirement for consistency of the present model to obtain: that choices can only be based on what agents know.

In this very general format the problem is rather intractable. Hence, to simplify the model while still capturing the essence of the issues at stake, we assume $T = 3$ and model gradual learning of types by choosing a very simple structure of shocks for each agent.

After the first period where productivities are identical, w, at the begining of each period an agent receives a shock to her productivity that may take one of two possible types H (for high) or L (for low).

We impose no restrictions on the probability distributions of shocks but independence across agents. A consequence is that ex-ante and ex-post distributions of types will coincide. There is, in this sense, no aggregate uncertainty

The evolution of information for an agent is, thus, given by the following filtration

$$\mathcal{F}_0 = \Omega;\ \mathcal{F}_1 = \{\{HH, HL\}, \{LH, LL\}\};\ \mathcal{F}_2 = \{HH, HL, LH, LL\}.$$

As we have already remarked, at each vertex of the event tree associated to the above filtration, there are only two states of the world: H and L. There are many different ways in which this filtration could be defined, this is but one which we take to be suitable for highlighting the most important aspects of the problem.

Thus, the expected utility of agents is

$$u\left(x^0, l^0\right) + \sum\nolimits_{i=H,L} \pi^i \left[u\left(x^i, l^i\right) + \sum\nolimits_{j=H,L} \pi^{j|i} u\left(x^{ji}, l^{ji}\right)\right], \tag{1}$$

where variables indexed with 0 are first period consumption and leisure - identical for all agents, since there is no heterogeneity 'ex ante' - π^i is the probability of an agent being of type i and $\pi^{j|i}$ is the probability of an agent being ji in the third period given that she was type i in the second period.

We call type i an agent with productivity $w \times i$ $(i = H, L)$ in the second period, and type ij an agent with productivity $w \times i \times j$ $(j = H, L)$ in the third. We use superscripts to denote that a certain associated with a given type. It is, then, clear that our assumptions imply $w^{HL} = w^{LH}$.

We start with some definitions and assumptions about the nature of agents' preferences. First define the type dependent second period indirect utility function

$$V(y, Y, w) \equiv u\left(y, \frac{Y}{w}\right),$$

where y denotes net income, Y the output produced and w the productivity of an agent - i.e., the amount of output produced per unit of time.

Let also

$$m(\cdot|ij) \equiv -\frac{\partial u\left(y, Y/w^{ij}\right)/\partial l}{\partial u\left(y, Y/w^{ij}\right)/\partial x}\frac{1}{w^{ij}} \equiv -\frac{V_Y\left(y, Y, w^{ij}\right)}{V_y\left(y, Y, w^{ij}\right)},$$

Whenever we consider a specific allocation we write

$$m\left(i'j'|ij\right) \equiv -\frac{V_Y\left(y^{i'j'}, Y^{i'j'}, w^{ij}\right)}{V_y\left(y^{i'j'}, Y^{i'j'}, w^{ij}\right)}$$

We adopt the usual assumption of single-crossing condition for preferences.
Assumption A: Bernoulli utility functions satisfy the Spence-Mirrlees - henceforth SM - condition $\partial m\left(\cdot\right)/\partial w < 0$.

We add to this a mild assumption of normality for leisure as well.
Assumption B: Leisure is a normal good.

We use the indirect utility functions to write (1) as

$$V\left(y, Y, w\right) + \sum\nolimits_{i=H,L}\left[\pi^i V\left(y^i, Y^i, w^i\right) + \sum\nolimits_{j=H,L}\pi^{j|i} V\left(y^{ji}, Y^{ji}, w^{ji}\right)\right] \quad (2)$$

A benevolent government maximizes expected utility of agents, taking into account the informational structure of the problem. The shocks are observed by each agent and only her. The government, on the other hand observes the output produced by each agent as well as her consumption in both periods.

The informational structure, along with a hypothesis of full-commitment, allows us to define the set of instruments that are available for the government to attain its goal of maximizing (2).

The description of the sequence of events in the model start with the first period labor supply and savings choices by an agent. At this moment, there is no informational asymmetry, hence there is no such a thing as deviating behavior. However anticipation of deviating behavior does play a role at this moment as we shall stress.

At the begining of the second period, the agents faces a shock in her productivity. After the first shock the government offers two contracts for the agents that specify a second period budget set, and two budget sets for third period, each associated to a specific second period announcement. A budget set is defined as follows:

$$\mathbb{B}^i \equiv \left\{\left(y, Y\right); \exists j \text{ with } \left(y, Y\right) = \left(y^{ij}, Y^{ij}\right)\right\}$$

where, $i = H, L, 0$.

We use y^{0j} to represent a type y^j in the second period, whenever there is no ambiguity.

Given our structure that allows for only two types at each node, there are two possible contracts from which the agent must choose, in the second period:

$$\left\{\left(y^H, Y^H\right), \left(\left(y^{HH}, Y^{HH}\right), \left(y^{HL}, Y^{HL}\right)\right)\right\} \text{ and}$$
$$\left\{\left(y^L, Y^L\right), \left(\left(y^{LH}, Y^{LH}\right), \left(y^{LL}, Y^{LL}\right)\right)\right\}$$

where the first part is the second period allocation and the last the third period budget sets.

In the third period the agent announces her type and receives the corresponding bundle, but only if the announced type is within the possible types contemplated in the contract.

To summarize, the time line for the model is as follows:

(y,Y) chosen	first signal observed	(y,Y) and 3^{rd} period contract chosen	type revealed	(y,Y) chosen

An announcement strategy is defined as a mapping from types to announcements. This mapping has to adapted to the filtration previously defined. That is $\sigma : \Omega \to \Omega$, such that σ_t is $\mathcal{F}_t$-measurable.

Agents maximize expected utility by choosing their preferred strategies. In fact, we can think of an agent, at each moment of time, looking ahead and choosing her announcement strategy knowing that conditional on her announcements she will face corresponding budget sets and allocations.

Because the objects of choice are strategies, what incentive compatibility - henceforth IC - constraints must guarantee, is that the strategy that prescribe truthful announcement, at every period, and for every realization, is the utility maximizing one.

Therefore, to advance in the characterization of this model we must understand what are the expected payoffs conditional on each strategy.

First, to reduce notational burden, we abuse notation a little bit by writing $V(i'j'|ij) \equiv V(y^{i'j'}, Y^{i'j'}, w^{ij})$ (resp. $V(i'|i) \equiv V(y^{i'}, Y^{i'}, w^{i})$, for second period types) to denote the momentary utility agent w^{ij} (resp. w^i) gets if she chooses the allocation intended for type $w^{i'j'}$ (resp. $w^{i'}$). However we use $V(ij) \equiv V(ij|ij)$, (resp. $V(i) \equiv V(i|i)$) to denote the allocation agent ij (resp. agent i) gets by truthfully announcing her type.

Then, following an announcement i', in the second period, and given a third period realization j, agent of type ij faces the problem of maximizing her utility by choice of a bundle within the budget set $\mathbb{B}^{i'}$.

We write $W(i'|ij)$ to represent the solution of this problem, that is

$$W(i'|ij) \equiv \max_{(y,Y)\in\mathbb{B}^{i'}} V\left(y, Y, w^{ij}\right) \tag{3}$$

We, then, define

$$\Upsilon(i) \equiv \pi^{H|i} W(i|iH) + \pi^{L|i} W(i|iL) \quad i = H, L, \text{ and}$$

$$\Upsilon(i'|i) \equiv \pi^{H|i} W(i'|iH) + \pi^{L|i} W(i'|iL) \quad i, i' = H, L$$

to denote, respectively, the expected utility that an agent of type i gets in the third period if she announces her true type in the second period, and the expected utility she gets if she announces another type, i'.

Bearing this notation in mind, we can state the government's program as that of maximizes, by choice of allocations, the objective function (1), subject to the second period incentive compatibility - henceforth, IC - constraints,

$$V\left(y^i, Y^i, w^i\right) + \Upsilon(i) \geq V\left(y^{i'}, Y^{i'}, w^i\right) + \Upsilon(i'|i) \quad i, i' = H, L, \tag{4}$$

the third period IC constraints,

$$V\left(y^{ij}, Y^{ij}, w^{ij}\right) \geq V\left(y^{ij'}, Y^{ij'}, w^{ij}\right), \quad \forall i, j, j', \tag{5}$$

and the resource constraint,

$$y - Y + \sum\nolimits_{i=H,L} \pi^{i}\left[y^{i} - Y^{i} + \sum\nolimits_{j=H,L} \pi^{j|i}\left(y^{ji} - Y^{ji}\right)\right] \leq 0. \tag{6}$$

To understand constraint (4), assume that the agent finds out that she is of type i. She knows what are the budget sets available in the third period given any of the two possible announcements. Whichever she chooses, she will, in the third period, choose the bundle within the corresponding budget set that maximizes her utility. This continuation payoff (or value function) is represented by $\Upsilon(i)$ for a truthful announcement and $\Upsilon(i'|i)$ for a false statement.

To induce truthful announcement the optimal allocations must be such that the sum of current period utility and continuation payoff from telling the truth must be at least as high as that of lying. This is what is guaranteed by (4). Furthermore, when third period comes, (5) makes sure that it is in the agents best interest to, once again, tell the truth.

We shall follow Cremer et al. (2001) in assuming that the usual - downward - constraints will bind. We do not know, as of this moment, whether the conditions imposed so far are enough to guarantee that this is necessary the case or whether other assumptions on preferences and/or the nature of shocks are needed.

With this assumption we are left with only three constraints. Two of them are in the third period,

$$V\left(y^{iH}, Y^{iH}, w^{iH}\right) \geq V\left(y^{iL}, Y^{iL}, w^{iH}\right), \ i = H, L,$$

and the other IC constraint is in the second period:

$$V\left(y^{H}, Y^{H}, w^{H}\right) + \Upsilon(H) \geq V\left(y^{L}, Y^{L}, w^{H}\right) + \Upsilon(L|H).$$

Related to these are the Lagrange multipliers μ^{i} $(i = H, L)$ and μ, respectivelly.

Notice also that, using the notation previously defined, and provided that the agent made a truthful announcement in second period, the third period IC constraints can also be written as $V(ij|ij) = W(i|ij)$ $(i = H, L)$.

We are now in a position to derive the optimal taxation scheme.

3 Labor Income Taxation

In characterizing an optimal tax system, of our highest interest are questions related to labor income marginal tax rates and inter-temporal distortion.

We start by discussing those labor-leisure distortions and leave it to section 4 the issues related to inter-temporal taxation.

3.1 Deriving Optimal Tax Formulae

We speak of an allocation as being distorted in the usual sense. That is, to denote a divergence between marginal rate of substitution and marginal rate of transformation. In the case of the labor/leisure choice this is the case whenever $m(ij) \neq 1$.

We then show that first period labor-leisure decisions are not distorted. In fact, one just have to notice that the first order conditions with respect to y and Y are, respectively,

$$V_y = \lambda \text{ and } V_Y = -\lambda \tag{7}$$

Because utility is inter-temporally separable *and* because savings are controlled by the government[3], distorting first period decisions will not help alleviating incentive problems.

3.1.1 Second Period Allocations

The first thing we show is that type H's labor-leisure choice is not distorted in the second period. In fact, the first order condition with respect to y^H is

$$V_y(H)\left(\pi^H + \mu\right) = \lambda \pi^H. \tag{8}$$

while the first order with respect to Y^H is

$$V_Y(H)\left(\pi^H + \mu\right) = -\lambda \pi^H. \tag{9}$$

Putting the two together gives us the result $m(H) = 1$. The decision is not distorted.

Low type, on the other hand, is distorted. To see this, just notice that the equivalent first order conditions are

$$\pi^L V_y(L) - \mu V_y(L|H) = \lambda \pi^L, \text{ and} \tag{10}$$

$$\pi^L V_Y(L) - \mu V_Y(L|H) = -\lambda \pi^L. \tag{11}$$

Hence,

$$m(L) - 1 = \frac{m(L|H) - 1}{\beta(L|H) + 1}, \tag{12}$$

where $\beta(L|H) = \lambda \pi^L \left(\mu V_y(L|H)\right)^{-1}$. Single crossing implies $m(L) > m(L|H)$. Because $\beta(H|L) > 0$, equation (12) can only be satisfied with $m(L) < 1$. That is a low type agent faces a positive marginal tax rate in the first period.

[3] The fact that government chooses - i.e., controls directly - agents savings is crucial for this result to obtain. In fact, da Costa (2003) shows how the result is changed if this assumption is not adopted.

3.1.2 Third Period Allocations

Let us now consider third period allocations. To do that we shall first investigate in further detail constraint (4). That is, it is necessary to provide a sharp charcetrization of these IC constraints for us to be able to understand how small changes in the different allocations affect incentives to deviate from prescribed behavior.

Because conditions (5) are satisfied, $W(H|HH) = V(HH)$ and $W(H|HL) = V(HL)$, hence,

$$\Upsilon(H) = \pi^{H|H} V(HH) + \pi^{L|H} V(HL).$$

As for $\Upsilon(L|H)$, we must first understand the off-equilibrium behavior of an agent of type H. This is accomplished with lemma 1, below.

Lemma 1 $\Upsilon(L|H) = \pi^{H|H} V(LH|HH) + \pi^{L|H} V(LH|HL)$.

What this lemma shows is that if at the second period a type H agent announces that she is of type L, she will always pick the allocation intended for type LH in the third period.

Making use of lemma 1 we can derive proposition 2 that characterizes last period's allocation.

Proposition 2 *In the third period, all agents but type HH, who is not distorted at the optimum, face a positive 'marginal tax rate'.*

Notice that we wrote 'marginal tax rates' under quotes for the usual reason of non-differentiability.

It is easy to understand why a type HH is not distorted at the optimum. Assume that she faces a positive marginal tax rate. Then, increasing both y^{HH} and Y^{HH} while preserving her utility, that is, moving along her indifference curve, increases revenue while not affecting her utility or incentives constraints. The same type of reasoning works for the case of a negative marginal tax rate.

As for the positive marginal tax rates on LH and LL, this is to be expected. In the last period, independently of what happened previously the allocations must be such as to induce a truthful announcement. The same kind of reasoning used in a static problem - e.g., Stiglitz (1982) - applies here, for both types.

Neither these results on the allocation of low types, nor the result on HH depend on the particular structure we have assumed. They hinge only on the particular set of binding constraints that are binding. They are to be valid whenever one can show that the binding constraints are the downward ones.

More interesting is the question of whether a high type in period 2 should be distorted. Notice that if one only consider the third period, this should not be the case. Given the utilitarian preferences of the government, it is always optimal to redistribute from a more productive type to a less productive one[4].

[4] With a slightly stronger assumption on preferences, namely, normality of consumption, it is always the case that there is some redistribution, either of income or of leisure that increases the social welfare function, independently on the number of types. See Brunner (1995).

Moreover, once third period arrives, and agents have truthfully declared their types, there is no one who envies the allocation intended for a type LH. Hence, only considering the equilibrium paths, there are no gains in distorting a type LH agent.

However, type LH is, indeed, distorted. The point here is that, though in equilibrium no one envies her allocation, type LH agent is to be distorted because of off-equilibrium choices by a high type.

To understand this result, assume that this is not the case. Then from the initial allocation produce a small change in the allocation intended for a type LH that preserves her utility. That is, $V_Y(LH)\,dY^{LH} + V_y(LH)\,dy^{LH} = 0 \Rightarrow dy^{LH} = m(LH)\,dY^{LH}$. Moreover, because, she faces a 0 marginal income tax rate, $m(LH) = 1$ and $dy^{LH} = dY^{LH}$, and the resource constraint is not affected. As for the high type who announces to be a low type, as argued before, ex-post, agents of type HL and LH are identical, so her utility is not affected. Recall, also, that from lemma 1 we have that type HH will also pick the allocation intended for LH. The impact on her utility is $dV(HH) = -V_y(HH)\left[m(HH) - m(HL)\right]dY^{LH} < 0$ for $dY^{LH} < 0$. This stems from the fact that the SM assumption guarantees that at any point, $m(HL) > m(HH)$. Therefore, we found a feasible reform that preserves all equilibrium expected utilities while relaxing IC constraints. The proposed optimal allocation cannot be optimal, then.

3.2 Further Characterizing the Optimal Allocations

Notwithstanding the fact that we have been able to derive some important properties of the allocations in section 3.1, to further gain insights about the optimal tax system, in this section, we examine two natural candidates for optimal tax system which we shall call: *isolated* and *memoryless* allocations.

The purpose of picking two natural candidates for optimal allocations - and checking whether any of them really characterizes the optimal tax structure of this economy - is to try to get a sharper characterization of the optimal tax system by analogy to well established results in public finance, provided that we can show that these candidates are, in fact, optimal.

Unfortunately, as we shall see, none of the natural candidates need to be optimal. In fact, we shall show that the *isolated* is never optimal. For the *memoryless* we show that it is not optimal in general and provide an example where it fails to be optimal.

However negative, these results are important in showing how complexity increases with the evolution of the information set.

3.2.1 Isolated

The first candidate is the allocation that treats separately every information vertex.

This allocation is built in two stages. First, one fixes the resouces available to the government at each node of the tree, B^i. With these resources budget sets are defined to maximize agents expected utilities given the usual IC constraints andthe resource constraints

$$y^{iH} + y^{iL} \leq Y^{iH} + Y^{iL} + B^{i} \quad i = H, L, 0. \tag{13}$$

The solution to these sub-problems define value functions for the government $\Phi\left(B^{i}\right)$.

Now the government can redistribute resources across types and periods optimally, provided that the transfer made from type H to type L does not lead to the violation of (4). That is,

$$\max V\left(y, Y\right) + \Phi\left(B\right) + \sum\nolimits_{i=H,L} \pi^{i} \Phi\left(B^{i}\right)$$

subject to

$$B + \sum\nolimits_{i=H,L} \pi^{i} \Phi\left(B^{i}\right) \leq Y - y$$

Beacuse every sub-problem is isolated in the sense that it is a traditional optimal taxation problem with resource constraints defined by (13), we can apply the well know results of optimal taxation to characterize some of its features.

In particular, we know that LH should not be distorted. But this contradicts proposition 2. Therefore, the optimal tax scheme cannot be of the isolated type.

3.2.2 Memoryless

The second candidate is what we call memoryless allocation. The reason why we give it this name is because it uses no information from the second period in drawing the allocation for the third one. We have not, however, restricted the resource constraint to things produced in each period alone.

By construction this allocation is implementable. In fact we have, in the third period,

$$\begin{aligned} V\left(HH\right) &= V\left(HL|HH\right) = V\left(LH|HH\right) \\ V\left(HL\right) &= V\left(LH\right) = V\left(LL|HL\right) = V\left(LL|LH\right) \end{aligned} \tag{14}$$

while in the second $V\left(H\right) = V\left(L|H\right)$. From the above equalities, and the fact that $V\left(\cdot|HL\right) = V\left(\cdot|LH\right)$, we have

$$\begin{aligned} \Upsilon\left(L|H\right) &= \pi^{H|H} V\left(LH|HH\right) + \pi^{L|H} V\left(LH|HL\right) \\ &= \pi^{H|H} V\left(HH\right) + \pi^{L|H} V\left(HL\right) = \Upsilon\left(H\right) \end{aligned}$$

So IC constraints (4) and (5) are satisfied.

Notice that, though conditions (14) are sufficient for the allocation to be implementable, they are not necessary. That is even if $V\left(L|H\right) > V\left(H\right)$, constraint (4) may be satisfied with $\Upsilon\left(L|H\right) < \Upsilon\left(H\right)$.

In general, there is no reason why one should think that the optimum would occur exactly with (14). In fact, we can easily derive conditions on the memoryless allocation for it not to be optimal.

Proposition 3 *If at the memoryless allocation,*

$$\frac{m(L)-m(L|H)}{m(LH)-m(LH|HH)} \neq \frac{\pi^{H|H}[1-m(L)]}{\pi^{H|L}[1-m(LH)]}$$

then the memoryless allocation is not optimal.

The conditions for a memoryless allocation to be optimal are very restrictive. However, they are not very intuitive since they are stated in terms of equilibrium allocations and not in terms of the primitives of our problem. We shall not attempt a complete characterization here. Instead, we provide next an example where the optimal allocation is not memoryless.

Example 4 *If* $\pi^i = \pi^{j|i} = 1/2$, *for* $i, j = H, L$, *then the memoryless contract is not optimal.*

There is nothing special about this example - which is fully developed in appendix B - but the fact that results are easier to prove under these assumptions on the probability distributions. What it shows, however, is that one should not expect optimal tax systems to be memoryless, in general.

4 Capital Income Taxation

When it comes to capital income taxation prescriptions, we should refer to two different sets of results that come up in the literature in recent years. On one hand, since the work of Chamley (1986), Judd (1987), and Lucas (1990), among others, it is known that in a Ramsey framework, it is typically optimal to set tax rate on capital income to zero, be it a steady state prescription or just a value to be satisfied on average.

On the other hand, repeated agency models, along the lines of Rogerson (1985) and, more recently, Golosov et al.. (2003), all find an inverse Euler equation rule that, in practice, represents taxing future consumption relative to present, or subsidizing today's consumption relative to the future.

We follow the second tradition of introducing informational asymmetries to the problem. More relevant to this particular problem, like them, we have a non-trivial evolution of the informational structure in our model economy, and this is the main driving force of the results we shall prove in this section 4.1.

4.1 General Preferences

Now we address the issue of capital income taxation policy in our framework. As we shall point out thorughout the section, it is the antecipation of off-equilibrium choices that generates most of the the results found in the more recent literature cited in the introduction to this section.

Back to the model, considering the previous discussion, and the particular setup we deal with, determining whether capital income is to be taxed or subsidized is equivalent to determining whether

$$V_y(i) \gtreqless \pi^{H|i} V_y(iH) + \pi^{L|i} V_y(iL) \text{ for } i = H, L, 0.$$

Let us start with first period. We can write the first order condition for y - as shown in (7) - as $V_y = \lambda\pi^H + \lambda\pi^L$. Then we can use (8) and (10) to write

$$V_y = \pi^H V_y(H) + \pi^L V_y(L) + \mu [V_y(H) - V_y(L|H)] \tag{15}$$

To tell whether savings ought to be taxed or subsidized, one would have to sign the term inside brackets in the right hand side of the above expression. To do this we need more information on preferences.

We have to compare a true high type with a high type pretending to be a low type. If allocations are monotonic, then we are talking about comparing someone with more income and labor with someone with less of both. If $u_{xl} \leq 0$ (that is the function defined in terms of leisure is supermodular) then we can prove that savings are increased. If $u_{xl} > 0$, however, we can't tell the sign.

After some algebraic manipulation (shown in appendix C for the reader's convenience), we have for type H,

$$\begin{aligned} V_y(H) &= V_y(HH)\pi^{H|H} + V_y(HL)\pi^{L|H} + \\ &\quad \frac{\mu^H}{\pi^H + \mu}[V_y(HH) - V_y(HL|HH)] \end{aligned} \tag{16}$$

The same type of reasoning used for the first period applies here to the case of a high type agent. Supermodularity, once again, suffices for the positive taxation on capital income result to obtain.

Now, for type L, things are not so simple. In fact,

$$\begin{aligned} V_y(L) &= \pi^{H|L} V_y(LH) + \pi^{L|L} V_y(LL) + \\ &\quad \frac{\mu^L}{\pi^L + \mu}[V_y(LH) - V_y(LL|LH)] - \\ &\quad \frac{\mu}{\pi^L + \mu}\{V_y(L|H) - [\pi^{H|H} V_y(LH|HH) + \pi^{L|H} V_y(LH|HL)] - \\ &\qquad V_y(L) - [\pi^{H|L} V_y(LH) + \pi^{L|L} V_y(LL)]\} \end{aligned} \tag{17}$$

As compared with (16), a new term appears in the right hand side. It is the difference between the intertemporal distortions faced by a true low type and a hight type announcing low.

Now it is not only the antecipation of deviating behavior that must be taken into account. When intertemporal allocations are distorted to prevent the antecipation of deviating behavior, this affects the incentives to tell the truth already at the second period. This must be taken into account when deciding on tax rules.

For the low type agent the same kind of reasoning cannot be used. One has also to take into account the behavior of a high type who mimics a low type.

4.2 Specializing Preferences

We shall now show how aditive separability delivers the taxation of capital income as the optimal policy.

That is, assume preferences are given by

$$u(x,l) \equiv \upsilon(x) - \zeta(l) \ \text{ or } V(y,Y,w) \equiv \upsilon(y) - \zeta(Y/w) \tag{18}$$

with $\upsilon', \zeta', \zeta'' > 0$ and $\upsilon'' < 0$.

First Period With preferences as in (18), expression (15) becomes

$$\upsilon'(y) = \pi^H \upsilon'\left(y^H\right) + \pi^L \upsilon'\left(y^L\right) + \mu\left[\upsilon'\left(y^H\right) - \upsilon'\left(y^L\right)\right]$$

We only need $\mu > 0$, and $\upsilon'\left(y^H\right) < \upsilon'\left(y^L\right)$. That is, monotonicity of allocations is sufficient to make subsidization of first period consumption optimal

High Type As for (16) it is easy to see that separability implies that

$$\begin{aligned}\upsilon'\left(y^H\right) &= \upsilon'\left(y^{HH}\right)\pi^{H|H} + \upsilon'\left(y^{HL}\right)\pi^{L|H} + \\ &\quad \frac{\mu^H}{\pi^H + \mu}\left[\upsilon'\left(y^{HH}\right) - \upsilon'\left(y^{HL}\right)\right].\end{aligned}$$

Once again, monotonicity of allocations is sufficient to guarantee that second period consumption ought to be subsidized (or savings ought to be taxed) relatively to third period consumption.

Low Type After some algebraic manipulation of (17) - also shown in appendix C - one derives the analogous condition for the low type

$$\begin{aligned}\upsilon'\left(y^L\right) &= \pi^{H|L}\upsilon'\left(y^{LH}\right) + \pi^{L|L}\upsilon'\left(y^{LL}\right) + \\ &\quad \frac{\mu^L + \pi^{L|L}\mu}{\pi^L + \mu}\left[\upsilon'\left(y^{LH}\right) - \upsilon'\left(y^{LL}\right)\right].\end{aligned}$$

When compared to the analogous expression for the high type, one extra term appears in the numerator of the term that multtiplies $\upsilon'\left(y^{LH}\right) - \upsilon'\left(y^{LL}\right)$. This extra term is exaclty to take into account deviating behavior in the second period, which is not important for a high type, where only antecipation of deviating behavior plays a role.

Hence, we get the result that, once again, savings are taxed (early consumption is subsidized).

Retirement It is also interesting to consider a retirement period. That is we introduce a forth period where nobody works.

The important thing about this is that form the third to the forth period the filtration is not changed. Is it still the case that we should tax returns? The answer is no.

To see this, one just have to change preferences a little bit to allow 'last period' utility to be represented by $\upsilon\left(y_1^{ij}\right)-\zeta\left(Y^{ij}/w^{ij}\right)+\upsilon\left(y_2^{ij}\right)$. It is, then, obvious that there is no gain in distinguishing y_1^{ij} from y_2^{ij}. This means that at the optimum savings from the third to the retirement period should not be taxed.

Notice that we do not get the result that retirements savings are not taxed. In fact, money invested in the first period to be redeemed at retirement should be taxed. The point is that marginal tax rates on return of those investments are necessarily lower than those of investments due in the third period.

More precisely, all investments made after all uncertainty is resolved must be free of taxes.

4.3 Discussion

Before moving on to the issue of supplementary commodity taxation, it is worth comparing the results herein with the findings of Golosov et al. (2003). To do this we shall attempt to use their variational approach inspired in Rogerson (1985) to our model.

Start from the optimum allocation and produce the following reform for type i allocation at the second period

$$\hat{y}^i \equiv y^i + \frac{\varepsilon}{V_y\left(i\right)},\ \hat{y}^{ij} = y^{ij} - \frac{\varepsilon}{V_y\left(ij\right)}.$$

It is clear that this reform has no impact on the utility of a i type. It costs to society

$$\frac{\varepsilon}{V_y\left(i\right)} - \sum_j \frac{\pi^{j|i}\varepsilon}{V_y\left(ij\right)}. \tag{19}$$

Suppose that there is no one who envies agent i. We would now want to argue that (19) ought to be zero, for there not to be a feasible improvement that contradicts our assertion that the initial allocation was optimal. The problem here is that feasibility is not guaranteed.

An agent of type $ij^{'}$ that envies the allocation intended for type ij - that is $V\left(ij|ij'\right) = V\left(ij'\right)$ - would have her off-equilibrium utility changed by $V_y\left(ij|ij'\right)$. Unless this is non-positive, the reform is not feasible. If it is, however, the cost of doing such a reform must be positive.

The important thing about this case is that we only have to check feasibility for the realized types. This is how we are able to find conditions under which taxing the high type (or investments in the first period) is optimal.

Things are even harder when there is a type i' that envies the allocation of type i. We must verify the effect of such a reform on the utility of a mimicker, i.e., a type i' who declares herself to be i.

In this case, $dV\left(i|i'\right) + d\Upsilon\left(i|i'\right) =$

$$V_y\left(i|i'\right)\frac{\varepsilon}{V_y\left(i\right)} - \sum\nolimits_{j'} \frac{W_y\left(i|i'j'\right)\pi\left(j'|i'\right)\varepsilon}{V_y\left(ij\right)}. \tag{20}$$

In general, this term is not 0. Suppose, then, that this term is negative for $\varepsilon > 0$, that is,

$$\frac{V_y(i|i')}{V_y(i)} < \sum_{j'} \frac{W_y(i|i'j')\,\pi(j'|i')}{V_y(ij)}.$$

Then, this reform must be costly, otherwise, one would be able to relax IC constraints. The inverse Euler equation becomes an inverse Euler inequality, which allows us to prove that $V_y(i) < \sum_j \pi^{j|i} V_y(ij)$. Notice, however, that we would still have to impose conditions on $V_y(ij|ij')$, as in the discussion of a non-envied type, for our argument to be valid.

The nice thing about the separable case is that $V_y(ij|ij') = V_y(ij) = v'\left(y^{ij}\right)$. Since the change in utility for all ij is $-\varepsilon$, total change in utility is $\varepsilon - \sum_j \pi^{j|i}\varepsilon = 0$. Moreover, for all types $i'j'$ there is a type ij such that $W_y(i|i'j') = V_y(ij)$. The same argument applies to show that $dV(i|i') + d\Upsilon(i|i') = 0$,

Hence, the value of (20) is 0. We can, then, proceed as Golosov et al. (2003) and argue that (19) must be 0 for all ε. An inverse Euler equation obtains.

The important lesson we draw from this discussion is that extending the result to more general preferences is not simple. Even if we impose conditions on cross derivatives to guarantee that 'ex-post' types do not find it optimal to deviate from prescribed behavior, the validity of (20) hinges upon third order cross derivatives which are not easy to interpret.

5 Taxing Goods

In this section we introduce multiple goods and find expressions for optimal taxes in this context. We then investigate the optimality of uniform taxation of goods.

Instead of considering a single consumption good, as we did in section 3 we assume in this section that besides leisure the agents consume n different goods. We shall represent a bundle of these goods with a vector $x \in \mathbb{R}^n$.

As for technology, we assume that one efficiency unit of labor is needed to produce a unit of any goods. That is technology is linear and we have chosen units as to make production prices equal to one for all goods.

The government is restricted to using linear taxes on goods. The idea is that the government cannot avoid trade - which we take to be costless - of these goods outside the production sector. If only trades between the consumption and production sector are under the government's control, then this makes non-linear taxation of goods unfeasible[5].

We keep however the assumption that inter-temporal trade can be controlled by the government. This is crucial for our results. In fact, in da Costa (2003) the consequences of removing this assumption for supplementary commodity taxation are discussed. Results derived here are not valid in that setup.

[5] See Guesnerie (1995) for a detailed discription of this mapping from the informational structure of the economy to the set of instruments available for the government.

Before deriving the results some notation will be needed. Define

$$V\left(q, y^{ij}, Y^{ij}, w^{ij}\right) \equiv \left\{ \begin{array}{c} \max_x u\left(x, Y^{ij}/w^{ij}\right) \\ \text{s.t.} \quad qx \leq y^{ij} \end{array} \right.$$

with corresponding (conditional) Marshallian demands $x\left(q, y^{ij}, Y^{ij}, w^{ij}\right)$.

Similarly, let

$$E\left(q, v^{ij}, Y^{ij}, w^{ij}\right) \equiv \left\{ \begin{array}{c} \min_x qx \\ \text{s.t.} \quad u\left(x, Y^{ij}/w^{ij}\right) \geq v^{ij} \end{array} \right.$$

with corresponding (conditional) Hicksian demands $h\left(q, y^{ij}, Y^{ij}, w^{ij}\right)$.

We could solve the modified program for the government right away. However we may simplify our derivations by rewriting the program in such a way as to have v as choice variable (instead of y) and re-defining the objective function

$$v + \sum_{i=H,L} \pi^i \left[v^i + \sum_{j=H,L} \pi^{j|i} v^{ij}\right].$$

The resource constraint must also be rewritten as

$$\iota h - Y + \sum_{i=H,L} \pi^i \left[\iota h\left(i\right) - Y^i + \sum_{j=H,L} \pi^{j|i}\left(\iota h\left(ij\right) - Y^{ji}\right)\right] \leq 0,$$

where ι is the n-dimensional vector of ones and $h\left(ij\right) \equiv h\left(q, y^{ij}, Y^{ij}, w^{ij}\right)$.

As for the IC constraints we first remark that

$$V\left(q, y^{i'j'}, Y^{i'j'}, w^{ij}\right) \equiv V\left(q, E\left(q, v^{i'j'}, Y^{i'j'}, w^{i'j'}\right), Y^{i'j'}, w^{ij}\right),$$

and use this identity for writing the IC constraints as follows:

$$v^{ij} \geq V\left(q, E\left(q, v^{i'j'}, Y^{i'j'}, w^{i'j'}\right), Y^{i'j'}, w^{ij}\right),$$

and corresponding expression for the first period IC constraints (4).

It turns out that this simplifies the derivation of our results a great deal. In fact, we just take the first order condition with respect to q^k to find

$$\begin{aligned} &\mu^H\left[x^k\left(HL|HH\right) - x^k\left(HL\right)\right] + \mu^L\left[x^k\left(LL|LH\right) - x^k\left(LL\right)\right] + \\ &\mu\left[x^k\left(L|H\right) - x^k\left(L\right)\right] + \mu\pi^{H|H}\left[x^k\left(LH|HH\right) - x^k\left(LH\right)\right] = \\ &\tau\nabla h^k + \sum_{i=H,L} \pi^i\left[\tau\nabla h^k\left(i\right) + \sum_{j=H,L} \pi^{j|i}\tau\nabla h^k\left(ij\right)\right] \end{aligned} \tag{21}$$

where $\left(\nabla h^k\right)' = \left(\partial h^k/\partial q^1, ..., \partial h^k/\partial q^n\right)$, and $\tau = \left(\tau^1, ..., \tau^n\right)$.

What these tax rules tell us is that, as in Mirrlees' (1976) analysis, the role of supplementary commodity taxation when an optimally designed tax schedule is available is to relax IC constraint by punishing off-equilibrium behavior.

There are two types of deviation that are relevant in the present problem. When third period arrives, an agent who receives a good shock - agents HH and LH - should be discouraged to announce a lower type. Hence, the way their consumption of good k differ from that of an agent they are claiming to be will signal their types. Taxing those goods whose consumption is higher for mimickers than for true low types, hurts deviators more than abiders and helps relax IC constraints. This is captured in the first two terms within brackets in the left hand side of (21).

In the second period there are two ways in which taxing goods may help. First, agents of type H who announce L, may have a different pattern of consumption than true low types - which is what the third term within brackets in the left hand side of (21) captures. Once again, observing consumption of goods that the mimicker consumes more relatively to a type she mimics will help infer that someone is lying.

But, there is another channel through which commodity taxation may help. As we have seen, from proposition 1 an agent of high type who claims to be of low type always chooses the bundle intended for type LH. Because we have assumed that $w^{LH} = w^{HL}$, whatever choices type LH makes in equilibrium, type HL will make off-equilibrium. Hence, consumption of this type will not help the planner. However, a HH type claiming to be type LH will, in general, change his consumption choice. Therefore, this type of deviation will signal her type.

Notice that in all cases it is the consumption of a more productive agent with a bundle of income. Hence, to tell whether the consumption of a good ought to be encouraged or discouraged[6] one would only have to find out if the demand for this good is positively or negatively related to leisure in the sense of Pollak (1969).

It is important to notice that neither this interpretation of taxation used for punishing the behavior of mimickers is not dependent on the specific pattern of binding IC constraints. Nor will it matter for the result in the next proposition, that shows that it is not always the case that taxing goods is helpful.

What does depend on our assumption about which constraints bind is the association of mimicking behavior with the specific pattern of conditional demands discussed in the previous paragraphs.

Proposition 5 *If utility is (weakly) separable between leisure and consumption goods, then uniform taxation is optimal.*

If utility is (weakly) separable between leisure and consumption then all terms in the left hand side of (21) vanishes. Proportional taxes, $\tau = \alpha\iota$, where α is a scalar and ι is as defined earlier, would do the job.

Notice that weak separability suffices. Golosov et al.. (2003) prove the Atkinson and Stiglitz result for the additive separable case. Their method of proof[7], however, is not suitable for a weaker form of separability.

[6] We use the term 'discourage', following Mirrlees (1976), to denote the linear approximation of the reduction in compensated demand induced by the tax system.

[7] Their method of proof is an adaptation for a self-selection model with a continuum of types of a procedure used by Rogerson (1985) in his classic repeated moral hazard article.

Moreover, it is never too much emphasizing that proposition 5 does not depend on the specific pattern of binding IC constraints. It is, in this sense, much more general than our simplified model may lead one to believe.

6 Conclusion

We derive in this paper optimal tax rules for a stylized model where agents gradually learn their productivities. The fact that the relevant filtration for the agent is non-trivial introduces a whole new set of issues to the optimal taxation problem.

The model, though recognizably very stylized, enlightens many of the issues at stake in more general setups and aids us in understanding the driving forces of key results found elsewhere. In terms of supplementary commodity taxation, knowing conditional demands is still key. In what we think is the most natural case, we show that goods positively related[8] to leisure ought to have their consumption discouraged in the sense of Mirrlees (1976). As a consequence, separability delivers Atkinson and Stiglitz's (1976) uniform taxation result.

We show that under additive separability between leisure and consumption capital income ought to be taxed.

As for implementation, taxes on savings must be made conditional on agents realized types. That means that an agent does not know the marginal tax rate she faces on her investment at the moment she decides to save. What she knows is how this tax rate varies with the productivity she will realize. In fact when taxes on return on capital are constrained to be measurable with respect to the previous period filtration - the key here is dependence with respect to agents current period's type - da Costa (2003) shows that many results are no longer valid. In particular, the uniform taxation breaks down.

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A Proofs

Proof of Lemma 1. Proving that $W(L|HL) = V(LH|HL)$ is rather simple. Given that $w^{LH} = w^{HL}$, the fact that the IC constraint (5) is satisfied for w^{LH} means that it is also satisfied for w^{HL}.That is an agent of type HL is intrinsically identical to an agent of type LH. Hence, he will pick the exact same allocation of a type LH.

We need only prove that an agent of type HH would prefer this allocation to the allocation intended for type LL. We show that if $V(LH|HL) = V(LL|HL)$ then $V(LH|HH) > V(LL|HH)$. This is a simple consequence of SM. Therefore $W(L|HH) = V(LH|HL)$ ■

Proof of Proposition 2. Next we write down all first order conditions for this problem. Starting with y^{HH} :

$$V_y(HH)\left[\pi^{HH}+\mu^H+\mu\pi^{H|H}\right]=\lambda\pi^{HH}, \tag{22}$$

then Y^{HH} :

$$V_Y(HH)\left[\pi^{HH}+\mu^H+\mu\pi^{H|H}\right]=-\lambda\pi^{HH}, \tag{23}$$

y^{HL} :

$$V_y(HL)\left[\pi^{HL}+\mu\pi^{H|H}\right]-\mu^H V_y(HL|HH)=\lambda\pi^{HL}, \tag{24}$$

Y^{HL} :

$$V_Y(HL)\left[\pi^{HL}+\mu\pi^{H|H}\right]-\mu^H V_Y(HL|HH)=-\lambda\pi^{HL}, \tag{25}$$

y^{LH} :

$$\begin{aligned}V_y(LH)\left[\pi^{LH}+\mu^L\right]-\mu[\pi^{H|H}V_y(LH|HH)+\\ \pi^{H|L}V_y(LH|HL)]=\lambda\pi^{LH},\end{aligned} \tag{26}$$

Y^{LH} :

$$\begin{aligned}V_Y(LH)\left[\pi^{LH}+\mu^L\right]-\mu[\pi^{H|H}V_Y(LH|HH)+\\ \pi^{H|L}V_Y(LH|HL)]=-\lambda\pi^{LH},\end{aligned} \tag{27}$$

y^{LL} :

$$V_y(LL)\,\pi^{LL}-\mu^L V_y(LL|LH)=\lambda\pi^{LL},\ \text{and,} \tag{28}$$

finally, Y^{LL} :

$$V_Y(LL)\,\pi^{LL}-\mu^L V_Y(LL|LH)=\lambda\pi^{LL}. \tag{29}$$

Dividing (23) by (22) gives, $m(HH)=1$, which shows that the marginal tax rate on type HH is 0.
As for type HL, the same procedure yields

$$m(HL)=\frac{\lambda\pi^{HL}-\mu^H V_Y(HL|HH)}{\lambda\pi^{HL}+\mu^H V_y(HL|HH)}$$

or,

$$m(HL)-1=\frac{m(HL|HH)-1}{\beta(HL|HH)+1}$$

where $\beta(HL|HH)$ is analogous to $\beta(L|H)$ in (12).The same type of reasoning leads to $m(HL)<1$.

We shall come back to this discussion later. For now, notice that dividing (27) by (26) and rearranging some terms we get an expression analogous to (12),

$$m\left(LH\right)-1=\frac{\hat{m}\left(L|H\right)-1}{1+\hat{\beta}\left(L|H\right)},$$

where

$$\hat{\beta}\left(L|H\right)\equiv\frac{\lambda\pi^{LH}}{\mu\left[\pi^{H|H}V_y\left(HH\right)+\pi^{H|L}V_y\left(HL\right)\right]},$$

and

$$\hat{m}\left(L|H\right)\equiv\alpha\left(L|H\right)m\left(LH|HH\right)+\left(1-\alpha\left(L|H\right)\right)m\left(LH|HL\right),$$

with

$$\alpha\left(L|H\right)=\frac{\pi^{H|H}V_y\left(LH|HH\right)}{\left[\pi^{H|H}V_y\left(LH|HH\right)+\pi^{H|L}V_y\left(LH|HL\right)\right]}.$$

For $\hat{\beta}\left(L|H\right)$ we only have to note that it is positive. As for $\hat{m}\left(L|H\right)$ it is a weighted average of $m\left(LH|HH\right)$ and $m\left(LH|HL\right)$. Because $m\left(LH|HL\right)=m\left(LH\right)$ and $m\left(LH|HH\right)<m\left(LH\right)$, from SM, $\hat{m}\left(L|H\right)<m\left(LH\right)$, and the same arguments used before apply here and allow us to see that $m\left(LH\right)<1$. As for type LL, we have

$$m(LL)-1=\frac{m\left(LL|LH\right)-1}{1+\beta\left(LL|LH\right)}.$$

We are tempted to say that the existence of a previous period has no bearing in the definition of the marginal tax rates that apply to type. However, the multiplier μ^L is affected by the existence of a previous period. Also, contrary to the one period case (or the high type allocation), the high type is already distorted, so $m\left(LL|LH\right)$ does not correspond, in general, to the same result one would obtain in the one period case. ■

Proof of Proposition 3. From the memoryless third period allocation we produce a reform in the optimal contract by changing the allocation offered for a type LH that preserves her utility. This implies,

$$dy^{LH}=m\left(LH\right)dY^{LH}$$

However, the effect on the utility of a high type how falsely announced to be of a low type, is

$$d\Upsilon\left(L|H\right)=\pi^{H|H}\left[m\left(LH\right)-m\left(LH|HH\right)\right]dY^{LH}$$

The problem is that this reform is not revenue preserving. In fact, since $m\left(LH\right)<1$ the change in revenue, $dR^3=\left[1-m\left(LH\right)\right]dY^{LH}$ will depend on the sign of dY^{LH}. To compensate for that we shall produce a welfare preserving change in the allocation offered for the low type at period 2. Once again, we have the condition $dy^L = m(L)\,dY^L$, with respective change in revenue, $dR^2 = [1 - m(L)]\,dY^L$.

This produces, however change in utility of a high type announcing L of

$$dV(L|H) = [m(L) - m(L|H)]\,dY^L.$$

Because we want the revenue to be kept constant, $dR^2 + dR^3 = 0 \Rightarrow$

$$\pi^{LH}[1 - m(LH)]\,dY^{LH} + \pi^L[1 - m(L)]\,dY^L = 0$$

Hence,

$$dY^L = -\pi^{H|L}\frac{1 - m(LH)}{1 - m(L)}dY^{LH}$$

The total effect on the utility of an agent of type H announcing L is simply, $dV(L|H) + d\Upsilon(L|H)$. Given all the conditions on the reform derived previously, the total effect on a high type is proportional to

$$-\left\{\frac{m(LH) - m(LH|HH)}{m(L) - m(L|H)} - \frac{\pi^{H|L}}{\pi^{H|H}}\frac{1 - m(LH)}{1 - m(L)}\right\}dY^{LH}.$$

So, unless the term within curly brackets is 0, there is a reform that is revenue preserving and relaxes the IC constraint while keeping equilibrium utility for a low type constant. ∎

Proof of Proposition 5. Just note that with (weak) separability, all terms in the left hand side of (21) vanish. Homogeneity of conditional Hicksian demands guarantees that with uniform taxation so does the right hand side. ∎

B Example 4

The first thing to notice here is that, if the optimal allocation is memoryless, then the constraints that are binding are exactly the ones we assumed to be binding. For the third period the result is a simple consequence of our assumptions on preferences and the governments redistributive motives.

For the second, just notice that

$$\begin{aligned}\Upsilon(L|H) &= \pi^{H|H}V(LH|HH) + \pi^{L|H}V(LH|HL)\\ &= \pi^{H|H}V(HL|HH) + \pi^{L|H}V(HL)\\ &= \pi^{H|H}V(HH) + \pi^{L|H}V(HL) = \Upsilon(H)\end{aligned}$$

Now, equation (27) can always be written as,

$$\begin{aligned}&V_Y(LH)\left[\pi^{LH} + \mu^H - \mu\pi^{H|L}\right] -\\ &\mu\pi^{H|H}V_Y(LH|HH) = -\lambda\pi^{LH},\end{aligned} \tag{30}$$

as a result of our assumption on productivities: $w^{HL} = w^{LH}$.
In the particular case of a memoryless contract, types LH and HL are not separated. Equation (25) can, in this case, be written as

$$V_Y(LH)\left[\pi^{HL} + \mu\pi^{H|H}\right] - \mu^H V_Y(LH|HH) = -\lambda\pi^{HL}, \tag{31}$$

Subtracting (31) from (30) we get

$$\begin{aligned} & V_Y(LH)\left[\left(\pi^{LH} - \pi^{HL}\right) + \mu^H - \mu\left(\pi^{H|L} + \pi^{H|H}\right)\right] - \\ & V_Y(HL|HH)\left[\mu\pi^{H|H} - \mu^H\right] = -\lambda\left(\pi^{LH} - \pi^{HL}\right), \end{aligned}$$

Now, assume $\pi^i = \pi^{j|i} = 1/2$, for $i, j = H, L$. Then,

$$V_Y(LH)\left[\mu^H - \mu\right] - \left(\frac{\mu}{2} + \mu^H\right) V_Y(HL|HH) = 0.$$

This implies

$$V_Y(LH) = \frac{\mu/2 - \mu^H}{\mu^H - \mu} V_Y(HL|HH)$$

Similarly, under the same assumptions, (26) and (24) become

$$V_y(LH)\left[\pi^{LH} + \mu^H - \pi^{H|L}\mu\right] - \mu\pi^{H|H} V_y(LH|HH) = \lambda\pi^{LH},$$

and

$$V_y(LH)\left[\pi^{HL} + \mu\pi^{H|H}\right] - \mu^H V_y(HL|HH) = \lambda\pi^{HL},$$

which implies

$$\begin{aligned} & V_y(LH)\left[\left(\pi^{LH} - \pi^{HL}\right) + \mu^H - \mu\left(\pi^{H|L} + \pi^{H|H}\right)\right] - \\ & V_y(LH|HH)\left[\mu\pi^{H|H} - \mu^H\right] = \lambda\left[\pi^{LH} - \pi^{HL}\right], \end{aligned}$$

Assume $\pi^i = \pi^{j|i} = 1/2$, for $i, j = H, L$. Then,

$$V_y(LH)\left[\mu^H - \mu\right] = \left[\mu/2 - \mu^H\right] V_y(LH|HH),$$

or

$$V_y(LH) = \frac{\mu/2 - \mu^H}{\mu^H - \mu} V_y(LH|HH)$$

Therefore, $m(LH) = m(LH|HH)$, which violates SM.

C Inter-temporal Allocations

We start by considering type H. Putting together her first order conditions along the lines of what was done, in section 4, for first period, one gets

$$\begin{aligned} V_y(H)\left(\pi^H+\mu\right) &= V_y(HH)\left[\pi^{HH}+\mu^H+\mu\pi^{H|H}\right]+ \\ & V_y(HL)\left[\pi^{HL}+\mu\pi^{H|H}\right]- \\ & \mu^H V_y(HL|HH). \end{aligned}$$

which may be rearranged into

$$\begin{aligned} V_y(H)\left(\pi^H+\mu\right) &= V_y(HH)\,\pi^{H|H}\left[\pi^H+\mu\right]+ \\ & V_y(HL)\,\pi^{L|H}\left[\pi^H+\mu\right]+ \\ & \mu^H\left[V_y(HH)-V_y(HL|HH)\right]. \end{aligned}$$

And finally,

$$\begin{aligned} V_y(H) &= V_y(HH)\,\pi^{H|H}+V_y(HL)\,\pi^{L|H}+ \\ & \frac{\mu^H}{\pi^H+\mu}\left[V_y(HH)-V_y(HL|HH)\right] \end{aligned} \tag{32}$$

A similar procedure for type L gives

$$\begin{aligned} \pi^L V_y(L)-\mu V_y(L|H) &= V_y(LH)\left[\pi^{LH}+\mu^L\right]- \\ & \mu[\pi^{H|H}V_y(LH|HH)+ \\ & \pi^{H|L}V_y(LH|HL)]+V_y(LL)\,\pi^{LL}- \\ & \mu^L V_y(LL|LH) \end{aligned}$$

which may be simplified a bit to read

$$\begin{aligned} V_y(L) &= \pi^{H|L}V_y(LH)+\pi^{L|L}V_y(LL)+ \\ & [V_y(LH)-V_y(LL|LH)]\,\mu^L/\pi^L-\{V_y(L|H)- \\ & [\pi^{H|H}V_y(LH|HH)+\pi^{H|L}V_y(LH|HL)]\}\mu/\pi^L. \end{aligned} \tag{33}$$

One last step,

$$\begin{aligned} & V_y(L)\left(\pi^L+\mu\right)-\mu V_y(L)=\left[\pi^{H|L}V_y(LH)+\pi^{L|L}V_y(LL)\right]\left(\pi^L+\mu\right) \\ & -\mu\left[\pi^{H|L}V_y(LH)+\pi^{L|L}V_y(LL)\right]+\mu^L\left[V_y(LH)-V_y(LL|LH)\right] \\ & -\mu\{V_y(L|H)-[\pi^{H|H}V_y(LH|HH)+\pi^{H|L}V_y(LH|HL)]\}\mu. \end{aligned}$$

When compared to the expression found for a high type we see an extra term that measures the inter-temporal distortion for an agent off the equilibrium path. That is, instead of providing directly the optimal distortion of a low type what the term $V_y(LH)-V_y(LL|LH)$ will give us is the relative distortion of a low type compared to the mimicker.

The problem here is that even if $V_y(LH)=V_y(LL|LH)$, it may be optimal to distort savings because distorting the decisions of a high type who announced to be a low type may help separating types in the second period.

C.1 Specializing Preferences

Now consider the effects of specilizing preferences by assuming separability.

C.1.1 First Period

With preferences defined by

$$u(x,l) \equiv \upsilon(x) - \zeta(l) \text{ or } V(y,Y,w) \equiv \upsilon(y) - \zeta(Y/w)$$

with $\upsilon', \zeta', \zeta'' > 0$ and $\upsilon'' < 0$. We have,

$$\begin{aligned} \upsilon'(y) &= \upsilon'\left(y^H\right)\pi^H + \upsilon'\left(y^L\right)\pi^L + \\ &\quad \mu\left[\upsilon'\left(y^H\right) - \upsilon'\left(y^L\right)\right] \end{aligned}$$

which shows that monotonicity of allocations is sufficient to make subsidization of first period consumption optimal.

C.1.2 High type

With separability, (32) becomes

$$\begin{aligned} \upsilon'\left(y^H\right) &= \upsilon'\left(y^{HH}\right)\pi^{H|H} + \upsilon'\left(y^{HL}\right)\pi^{L|H} + \\ &\quad \frac{\mu^H}{\pi^H + \mu}\left[\upsilon'\left(y^{HH}\right) - \upsilon'\left(y^{HL}\right)\right] \end{aligned}$$

C.1.3 Low Type

With separability, (17) becomes

$$\begin{aligned} \upsilon'\left(y^L\right) &= \pi^{H|L}\upsilon'\left(y^{LH}\right) + \pi^{L|L}\upsilon'\left(y^{LL}\right) + \\ &\quad \frac{\mu^L}{\pi^H + \mu}\left[\upsilon'\left(y^{LH}\right) - \upsilon'\left(y^{LL}\right)\right] - \\ &\quad \frac{\mu}{\pi^L + \mu^L}\left\{\upsilon'\left(y^L\right) - \left[\pi^{H|H}\upsilon'\left(y^{LH}\right) + \pi^{L|H}\upsilon'\left(y^{LH}\right)\right]\right\} + \\ &\quad \frac{\mu}{\pi^L + \mu^L}\left\{\upsilon'\left(y^L\right) - \left[\pi^{H|L}\upsilon'\left(y^{LH}\right) + \pi^{L|L}\upsilon'\left(y^{LH}\right)\right]\right\} \end{aligned}$$

or

$$\begin{aligned} \upsilon'\left(y^L\right) &= \pi^{H|L}\upsilon'\left(y^{LH}\right) + \pi^{L|L}\upsilon'\left(y^{LL}\right) + \\ &\quad \frac{\mu^L}{\pi^L + \mu}\left[\upsilon'\left(y^{LH}\right) - \upsilon'\left(y^{LL}\right)\right] + \\ &\quad \frac{\mu\pi^{L|L}}{\pi^L + \mu^L}\left[\upsilon'\left(y^{LH}\right) - \upsilon'\left(y^{LL}\right)\right] \end{aligned}$$

where we just used the fact that $\pi^{H|i} + \pi^{L|i} = 1$, $(i = H, L)$.

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